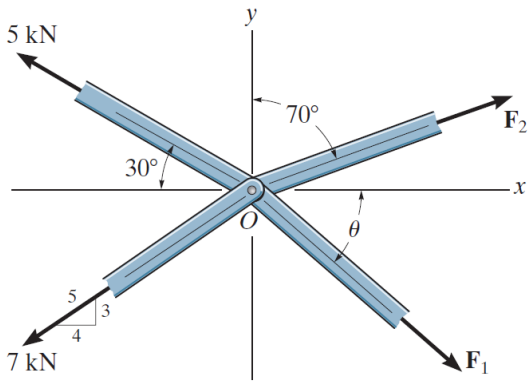
Problem 3-1

The members of a truss are pin connected at joint O. Determine the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ for equilibrium. Set $\theta = 60°$.



Probs. 3–1/2

Solution

Begin by finding α, the angle that the 7 kN force makes with the x-axis.

$$\tan\alpha = \frac{3}{4} \quad \rightarrow \quad \alpha = \tan^{-1}\left(\frac{3}{4}\right) \approx 36.9°$$

In order for the truss to be in equilibrium, the sum of the forces in each direction must be zero.

$$\sum F_x = 0: \qquad F_2\sin 70° + F_1\cos 60° - 5000\cos 30° - 7000\cos\alpha = 0$$

$$\sum F_y = 0: \qquad F_2\cos 70° - F_1\sin 60° + 5000\sin 30° - 7000\sin\alpha = 0$$

Bring the terms without F_1 and F_2 to the right side.

$$F_2\sin 70° + F_1\cos 60° = 5000\cos 30° + 7000\cos\alpha \tag{1}$$

$$F_2\cos 70° - F_1\sin 60° = 7000\sin\alpha - 5000\sin 30° \tag{2}$$

To eliminate F_2, multiply both sides of equation (1) by $\cos 70°$ and multiply both sides of equation (2) by $\sin 70°$.

$$F_2\sin 70°\cos 70° + F_1\cos 60°\cos 70° = (5000\cos 30° + 7000\cos\alpha)\cos 70°$$

$$F_2\cos 70°\sin 70° - F_1\sin 60°\sin 70° = (7000\sin\alpha - 5000\sin 30°)\sin 70°$$

Subtract the respective sides and solve for F_1.

$$F_1(\cos 60°\cos 70° + \sin 60°\sin 70°) = (5000\cos 30° + 7000\cos\alpha)\cos 70° - (7000\sin\alpha - 5000\sin 30°)\sin 70°$$

$$F_1 = \frac{(5000\cos 30° + 7000\cos\alpha)\cos 70° - (7000\sin\alpha - 5000\sin 30°)\sin 70°}{\cos 60°\cos 70° + \sin 60°\sin 70°} \approx 1.83\times 10^3 \text{ N}$$

To eliminate F_1 instead, multiply both sides of equation (1) by $\sin 60^\circ$ and multiply both sides of equation (2) by $\cos 60^\circ$.

$$F_2 \sin 70^\circ \sin 60^\circ + F_1 \cos 60^\circ \sin 60^\circ = (5000 \cos 30^\circ + 7000 \cos \alpha) \sin 60^\circ$$

$$F_2 \cos 70^\circ \cos 60^\circ - F_1 \sin 60^\circ \cos 60^\circ = (7000 \sin \alpha - 5000 \sin 30^\circ) \cos 60^\circ$$

Add the respective sides and solve for F_2.

$$F_2(\sin 70^\circ \sin 60^\circ + \cos 70^\circ \cos 60^\circ) = (5000 \cos 30^\circ + 7000 \cos \alpha) \sin 60^\circ + (7000 \sin \alpha - 5000 \sin 30^\circ) \cos 60^\circ$$

$$F_2 = \frac{(5000 \cos 30^\circ + 7000 \cos \alpha) \sin 60^\circ + (7000 \sin \alpha - 5000 \sin 30^\circ) \cos 60^\circ}{\sin 70^\circ \sin 60^\circ + \cos 70^\circ \cos 60^\circ} \approx 9.60 \times 10^3 \text{ N}$$